

AMENDMENTS TO CLAIMS

Claim 1 (currently amended): A digital signature cryptographic method comprising:

supplying a set S1 of k polynomial functions as a public-key, the set S1 including the functions $P_1(x_1, \dots, x_{n+v}, y_1, \dots, y_k), \dots, P_k(x_1, \dots, x_{n+v}, y_1, \dots, y_k)$, where k, v, and n are integers, x_1, \dots, x_{n+v} are n+v variables of a first type, y_1, \dots, y_k are k variables of a second type, and the set S1 is obtained by applying a secret key operation on a set S2 of k polynomial functions $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$ where a_1, \dots, a_{n+v} are n+v variables which include a set of n "oil" variables a_1, \dots, a_n , and a set of v "vinegar" variables a_{n+1}, \dots, a_{n+v} , the supplying comprising selecting the number v of "vinegar" variables to be greater than the number n of "oil" variables;

providing a message to be signed;

applying a hash function on the message to produce a series of k values b_1, \dots, b_k ;

C 1 substituting the series of k values b_1, \dots, b_k for the variables y_1, \dots, y_k of the set S2 respectively to produce a set S3 of k polynomial functions $P''_1(a_1, \dots, a_{n+v}), \dots, P''_k(a_1, \dots, a_{n+v})$;

selecting v values $a'_{n+1}, \dots, a'_{n+v}$ for the v "vinegar" variables a_{n+1}, \dots, a_{n+v} ;

solving a set of equations $P''_1(a_1, \dots, a_n, a'_{n+1}, \dots, a'_{n+v})=0, \dots, P''_k(a_1, \dots, a_n, a'_{n+1}, \dots, a'_{n+v})=0$ to obtain a solution for a'_1, \dots, a'_n ; and

applying the secret key operation to transform a'_1, \dots, a'_{n+v} to a digital signature e_1, \dots, e_{n+v} .

Claim 2 (previously presented): A method according to claim 1 and also comprising verifying the digital signature.

Claim 3 (previously presented): A method according to claim 2 and wherein said verifying comprises:

obtaining the signature e_1, \dots, e_{n+v} , the message, the hash function and the public key;

applying the hash function on the message to produce the series of k values b_1, \dots, b_k ; and

verifying that the equations $P_1(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0, \dots, P_k(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0$ are satisfied.

Claim 4 (previously presented): A method according to claim 1 and wherein the method comprises an HFEV scheme and the set $S2$ comprises a set $f(a)$ of k polynomial functions of the HFEV scheme.

Claim 5 (previously presented): A method according to claim 1 and wherein the method comprises a UOV scheme and the set $S2$ comprises a set S of k polynomial functions of the UOV scheme.

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Claim 6 (canceled)

Claim 7 (previously presented): A method according to claim 1 and wherein v is selected such that q^v is greater than 2^{32} , where q is the number of elements of a finite field K over which the sets $S1$, $S2$ and $S3$ are provided.

Claim 8 (previously presented): A method according to claim 1 and wherein said supplying comprises obtaining the set $S1$ from a subset $S2'$ of k polynomial functions of the set $S2$, the subset $S2'$ being characterized in that all coefficients of components involving any of the y_1, \dots, y_k variables in the k polynomial functions $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$ are zero, and the number v of "vinegar" variables is greater than the number n of "oil" variables.

Claim 9 (previously presented): A method according to claim 8 and wherein the set $S2$ comprises a set S of k polynomial functions of a UOV scheme, and the number v of "vinegar" variables is selected to satisfy one of the following conditions:

- (a) for each characteristic p other than 2 of a field K in an "Oil and Vinegar" scheme of degree 2, v satisfies the inequality $q^{(v-n)-1} * n^4 > 2^{40}$, where K is a finite field over which the sets $S1$, $S2$ and $S3$ are provided,
- (b) for $p = 2$ in an "Oil and Vinegar" scheme of degree 3, v is greater than $n*(1 + \sqrt{3})$ and less than or equal to $n^3/6$, and
- (c) for each p other than 2 in an "Oil and Vinegar" scheme of degree 3, v is greater than n and less than or equal to n^4 .

Claim 10 (previously presented): A method according to claim 8 and wherein the set $S2$ comprises a set S of k polynomial functions of a UOV scheme, and the number v of "vinegar" variables is selected to satisfy the inequalities $v < n^2$ and $q^{(v-n)-1} * n^4 > 2^{40}$ for a characteristic $p=2$ of a field K in an "Oil and Vinegar" scheme of degree 2, where K is a finite field over which the sets $S1$, $S2$ and $S3$ are provided and q is the number of elements of K .

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Claim 11 (original): A method according to claim 1 and wherein said secret key operation comprises a secret affine transformation s on the $n+v$ variables a_1, \dots, a_{n+v} .

Claim 12 (original): A method according to claim 4 and wherein said set $S2$ comprises an expression including k functions that are derived from a univariate polynomial.

Claim 13 (original): A method according to claim 12 and wherein said univariate polynomial includes a univariate polynomial of degree less than or equal to 100,000.

Claim 14 (original): A cryptographic method for verifying the digital signature of claim 1, the method comprising:

obtaining the signature e_1, \dots, e_{n+v} , the message, the hash function and the public key;

applying the hash function on the message to produce the series of k values b_1, \dots, b_k ; and

verifying that the equations $P_1(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0, \dots, P_k(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0$ are satisfied.

Claim 15 (previously presented): In an "Oil and Vinegar" signature method, an improvement comprising using more "vinegar" variables than "oil" variables.

Claim 16 (previously presented): A method according to claim 15 and wherein a number v of "vinegar" variables is selected to satisfy one of the following conditions:

- C¹
- (a) for each characteristic p other than 2 of a field K and for a degree 2 of the "Oil and Vinegar" signature method, v satisfies the inequality $q^{(v-n)-1} * n^4 > 2^{40}$, where n is a number of "oil" variables, K is a finite field from which the n "oil" variables and the v "vinegar" variables are selected, and q is the number of elements of K ,
 - (b) for $p = 2$ and for a degree 3 of the "Oil and Vinegar" signature method, v is greater than $n * (1 + \sqrt{3})$ and less than or equal to $n^3/6$, and
 - (c) for each p other than 2 and for a degree 3 of the "Oil and Vinegar" signature method, v is greater than n and less than or equal to n^4 .

Claim 17 (previously presented): A method according to claim 15 and wherein a number v of "vinegar" variables is selected to satisfy the inequalities $v < n^2$ and $q^{(v-n)-1} * n^4 > 2^{40}$ for a characteristic $p=2$ of a field K in an "Oil and Vinegar" scheme of degree 2, where n is a number of "oil" variables, K is a finite field from which the n "oil" variables and the v "vinegar" variables are selected, and q is the number of elements of K .

Claim 18 (currently amended): A signature generator comprising:

a signature input receiver operative to receive a set S1 of k polynomial functions as a public-key and a message to be signed, the set S1 including the functions $P_1(x_1, \dots, x_{n+v}, y_1, \dots, y_k), \dots, P_k(x_1, \dots, x_{n+v}, y_1, \dots, y_k)$, where k, v, and n are integers, x_1, \dots, x_{n+v} are n+v variables of a first type, y_1, \dots, y_k are k variables of a second type, and the set S1 is obtained by applying a secret key operation on a set S2 of k polynomial functions $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$, where a_1, \dots, a_{n+v} are n+v variables which include a set of n "oil" variables a_1, \dots, a_n , and a set of v "vinegar" variables a_{n+1}, \dots, a_{n+v} and the number v of "vinegar" variables is greater than the number n of "oil" variables; and

a signature processor operatively associated with the signature input receiver and operative to perform the following operations:

to apply a hash function on the message to produce a series of k values b_1, \dots, b_k ,

to substitute the series of k values b_1, \dots, b_k for the variables y_1, \dots, y_k of the set S2 respectively to produce a set S3 of k polynomial functions $P''_1(a_1, \dots, a_{n+v}), \dots, P''_k(a_1, \dots, a_{n+v})$,

to select v values $a'_{n+1}, \dots, a'_{n+v}$ for the v "vinegar" variables a_{n+1}, \dots, a_{n+v} ;

to solve a set of equations $P''_1(a_1, \dots, a_n, a'_{n+1}, \dots, a'_{n+v})=0, \dots, P''_k(a_1, \dots, a_n, a'_{n+1}, \dots, a'_{n+v})=0$ to obtain a solution for a'_1, \dots, a'_n ; and

to apply the secret key operation to transform a'_1, \dots, a'_{n+v} into a digital signature e_1, \dots, e_{n+v} .

Claim 19 (previously presented): Apparatus according to claim 18 and also comprising a signature verifier operatively associated with the signature processor and operative to verify the digital signature.

Claim 20 (previously presented): Apparatus according to claim 19 and wherein said signature verifier is operative to verify the digital signature by performing the following operations:

obtaining the signature e_1, \dots, e_{n+v} , the message, the hash function and the public key;

applying the hash function on the message to produce the series of k values b_1, \dots, b_k ; and

verifying that the equations $P_1(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0, \dots, P_k(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0$ are satisfied.

Claim 21 (previously presented): Apparatus according to claim 18 and wherein the signature processor is operative to perform an HFEV scheme, and the set S_2 comprises a set $f(a)$ of k polynomial functions of the HFEV scheme.

Claim 22 (previously presented): Apparatus according to claim 18 and wherein the signature processor is operative to perform a UOV scheme, and the set S_2 comprises a set S of k polynomial functions of the UOV scheme.

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Claim 23 (canceled)

Claim 24 (previously presented): Apparatus according to claim 18 and wherein v is selected such that q^v is greater than 2^{32} , where q is the number of elements of a finite field K over which the sets S_1 , S_2 and S_3 are provided.

Claim 25 (previously presented): Apparatus according to claim 18 and wherein the set S_1 is obtained from a subset S_2' of k polynomial functions of the set S_2 , the subset S_2' being characterized in that all coefficients of components involving any of the y_1, \dots, y_k variables in the k polynomial functions $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$ are zero, and the number v of "vinegar" variables is greater than the number n of "oil" variables.

Claim 26 (previously presented): Apparatus according to claim 25 and wherein the set S_2 comprises a set S of k polynomial functions of a UOV scheme, and the

number v of “vinegar” variables is selected to satisfy one of the following conditions:

- (a) for each characteristic p other than 2 of a field K in an “Oil and Vinegar” scheme of degree 2, v satisfies the inequality $q^{(v-n)-1} * n^4 > 2^{40}$, where K is a finite field over which the sets $S1$, $S2$ and $S3$ are provided,
- (b) for $p = 2$ in an “Oil and Vinegar” scheme of degree 3, v is greater than $n*(1 + \sqrt{3})$ and less than or equal to $n^3/6$, and
- (c) for each p other than 2 in an “Oil and Vinegar” scheme of degree 3, v is greater than n and less than or equal to n^4 .

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Claim 27 (currently amended): Apparatus according to claim 25 and wherein the set $S2$ comprises a set S of k polynomial functions of a UOV scheme, and the number v of “vinegar” variables is selected to satisfy the inequalities $v < n^2$ and $q^{(v-n)-1} * n^4 > 2^{40}$ for a characteristic $p=2$ of a field K in an “Oil and Vinegar” scheme of degree 2, $[[2, _]]$ where K is a finite field over which the sets $S1$, $S2$ and $S3$ are provided and q is the number of elements of K .

Claim 28 (previously presented): Apparatus according to claim 18 and wherein said secret key operation comprises a secret affine transformation s on the $n+v$ variables a_1, \dots, a_{n+v} .

Claim 29 (previously presented): Apparatus according to claim 21 and wherein said set $S2$ comprises an expression including k functions that are derived from a univariate polynomial.

Claim 30 (previously presented): Apparatus according to claim 29 and wherein said univariate polynomial includes a univariate polynomial of degree less than or equal to 100,000.

Claim 31 (previously presented): A signature verifier for verifying the digital signature generated by the signature generator of claim 18, the signature verifier comprising a verifier processor operative to perform the following operations:

to obtain the signature e_1, \dots, e_{n+v} , the message, the hash function and the public key via the signature input receiver;

to apply the hash function on the message to produce the series of k values b_1, \dots, b_k ; and

to verify that the equations $P_1(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0, \dots, P_k(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0$ are satisfied.

Claim 32 (previously presented): In an "Oil and Vinegar" signature generating apparatus an improvement characterized in that the "Oil and Vinegar" signature generating apparatus is operative to use more "vinegar" variables than "oil" variables.

Claim 33 (currently amended): An "Oil and Vinegar" signature generating apparatus according to claim 32 and wherein a number v of "vinegar" variables is selected to satisfy one of the following conditions:

(a) for each characteristic p other than 2 of a field K and for a degree 2 of an "Oil and Vinegar" signature method, v satisfies the inequality $q^{(v-n)-1} * n^4 > 2^{40}$, where n is a number of "oil" variables, K is a finite field from which the n "oil" variables and the v "vinegar" variables are selected, and q is the number of elements of K ,

(b) for $p = 2$ and for a degree 3 of the "Oil and Vinegar" signature method, v is greater than $n * (1 + \sqrt{3})$ and less than or equal to $n^3/6$, and

(c) for each p other than 2 and for a degree 3 of the "Oil and Vinegar" signature method, v is greater than n and less than or equal to n^4 .

Claim 34 (previously presented): An "Oil and Vinegar" signature generating apparatus according to claim 32 and wherein a number v of "vinegar" variables is

selected to satisfy the inequalities $v < n^2$ and $q^{(v-n)-1} * n^4 > 2^{40}$ for a characteristic $p=2$ of a field K in an "Oil and Vinegar" scheme of degree 2, where n is a number of "oil" variables, K is a finite field from which the n "oil" variables and the v "vinegar" variables are selected, and q is the number of elements of K .

Claim 35 (currently amended): A digital signature comprising:

a signature e_1, \dots, e_{n+v} generated by processing a set $S1$ of k polynomial functions provided as a public-key and a message to be signed, where the set $S1$ includes functions $P_1(x_1, \dots, x_{n+v}, y_1, \dots, y_k), \dots, P_1(\cancel{x_1, \dots, x_{n+v}}, \cancel{y_1, \dots, y_k}), \dots, P_k(x_1, \dots, x_{n+v}, y_1, \dots, y_k)$, where k, v , and n are integers, x_1, \dots, x_{n+v} are $n+v$ variables of a first type, y_1, \dots, y_k are k variables of a second type, and the set $S1$ is obtained by applying a secret key operation on a set $S2$ of k polynomial functions $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$ where a_1, \dots, a_{n+v} are $n+v$ variables which include a set of n "oil" variables a_1, \dots, a_n , and a set of v "vinegar" variables a_{n+1}, \dots, a_{n+v} , and the number v of "vinegar" variables is greater than the number n of "oil" variables, so that a hash function applied on the message to produce a series of k values b_1, \dots, b_k that are substituted for the variables y_1, \dots, y_k of the set $S2$ respectively to produce a set $S3$ of k polynomial functions $P''_1(a_1, \dots, a_{n+v}), \dots, P''_k(a_1, \dots, a_{n+v})$ and v values $a'_{n+1}, \dots, a'_{n+v}$ that are selected for the v "vinegar" variables a_{n+1}, \dots, a_{n+v} , enable to solve a set of equations $P''_1(a_1, \dots, a_n, a'_{n+1}, \dots, a'_{n+v})=0, \dots, P''_k(a_1, \dots, a_n, a'_{n+1}, \dots, a'_{n+v})=0$ to obtain a solution for a'_1, \dots, a'_n , and application of the secret key operation transforms a'_1, \dots, a'_{n+v} into the digital signature e_1, \dots, e_{n+v} .

Claim 36 (previously presented): A digital signature produced by the method of claim 1.

Claim 37 (previously presented): A method according to claim 1 and wherein said supplying comprises obtaining the set $S1$ from a subset $S2'$ of k polynomial functions of the set $S2$, the subset $S2'$ being characterized in that all coefficients of components involving orders higher than 1 of any of the n "oil" variables a_1, \dots, a_n

and coefficients of components involving multiplication of two or more of the n "oil" variables a_1, \dots, a_n in the k polynomial functions $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$ are zero, and the number v of "vinegar" variables is greater than the number n of "oil" variables.

Claim 38 (previously presented): A method according to claim 37 and wherein the set S_2 comprises a set S of k polynomial functions of a UOV scheme, and the number v of "vinegar" variables is selected to satisfy one of the following conditions:

- (a) for each characteristic p other than 2 of a field K in an "Oil and Vinegar" scheme of degree 2, v satisfies the inequality $q^{(v-n)-1} * n^4 > 2^{40}$, where K is a finite field over which the sets S_1 , S_2 and S_3 are provided,
- (b) for $p = 2$ in an "Oil and Vinegar" scheme of degree 3, v is greater than $n * (1 + \sqrt{3})$ and less than or equal to $n^3/6$, and
- (c) for each p other than 2 in an "Oil and Vinegar" scheme of degree 3, v is greater than n and less than or equal to n^4 .

Claim 39 (previously presented): A method according to claim 37 and wherein the set S_2 comprises a set S of k polynomial functions of a UOV scheme, and the number v of "vinegar" variables is selected to satisfy the inequalities $v < n^2$ and $q^{(v-n)-1} * n^4 > 2^{40}$ for a characteristic $p=2$ of a field K in an "Oil and Vinegar" scheme of degree 2, where K is a finite field over which the sets S_1 , S_2 and S_3 are provided and q is the number of elements of K .

Claim 40 (previously presented): Apparatus according to claim 18 and wherein the set S_1 is obtained from a subset S_2' of k polynomial functions of the set S_2 , the subset S_2' being characterized in that all coefficients of components involving orders higher than 1 of any of the n "oil" variables a_1, \dots, a_n and coefficients of components involving multiplication of two or more of the n "oil" variables a_1, \dots, a_n in the k polynomial functions $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$ are

zero, and the number v of “vinegar” variables is greater than the number n of “oil” variables.

Claim 41 (previously presented): Apparatus according to claim 40 and wherein the set S_2 comprises a set S of k polynomial functions of a UOV scheme, and the number v of “vinegar” variables is selected to satisfy one of the following conditions:

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- (a) for each characteristic p other than 2 of a field K in an “Oil and Vinegar” scheme of degree 2, v satisfies the inequality $q^{(v-n)-1} * n^4 > 2^{40}$, where K is a finite field over which the sets S_1 , S_2 and S_3 are provided,
 - (b) for $p = 2$ in an “Oil and Vinegar” scheme of degree 3, v is greater than $n * (1 + \sqrt{3})$ and less than or equal to $n^3/6$, and
 - (c) for each p other than 2 in an “Oil and Vinegar” scheme of degree 3, v is greater than n and less than or equal to n^4 .

Claim 42 (previously presented): Apparatus according to claim 40 and wherein the set S_2 comprises a set S of k polynomial functions of a UOV scheme, and the number v of “vinegar” variables is selected to satisfy the inequalities $v < n^2$ and $q^{(v-n)-1} * n^4 > 2^{40}$ for a characteristic $p=2$ of a field K in an “Oil and Vinegar” scheme of degree 2, where K is a finite field over which the sets S_1 , S_2 and S_3 are provided and q is the number of elements of K .
